SUBJECT: Notes on Coding Theory

Quantum Corporation, Irvine, 2013.

Incomplete Beta Function for computing the CDF of Binomial Distribution

Let us assume that bit error probability at the output of the detector is uniformly random and independent from one bit position to another in NRZI format. Note that this assumption on NRZ output does not necessarily lead to the same assumption for NRZI bits due to pre-coding operation that introducing dependency.

Let the NRZI bit error probability be ρ. Assuming all error happen uniformly randomly and independent of each other, a byte error probability can be computed as

$$P_{byte} = 1 - (1 - \rho)^8 \approx 8\rho \text{ for } \rho << 1 \tag{1}$$

Since a CW is uncorrectable when we have 6 or more byte errors, the probability of having an uncorrectable C1 CW is given by

$$P_{C1,uncorr.} = \sum_{i=6}^{240} \binom{240}{i} P_{byte}^i (1 - P_{byte})^{240-i} = I_{P_{byte}}(6, 240 - 5) \tag{2}$$

where P_{byte} is the probability of independent byte errors and $I_x(a,b) = B(x;a,b)/B(a,b)$ and $B(x;a,b)$ is called incomplete beta function and given by

$$B(x;a,b) = \int_0^x t^{a-1}(1-t)^{b-1}dt \tag{3}$$

and when $x = 1$ we have the complete beta function i.e., $B(a,b) = B(1;a,b)$. Beta functions are numerically more stable to compute than the CDF of binomial distribution. See Matlab functions betainc.m and binocdf.m for reference.

For example, $betainc(P_{byte}, 6, 240 - 5)$ computes the sum in Eqn. (2). From basic coding theory, puncturing/shortening the RS code results in another maximum distance separable RS code. Therefore, if b parity bytes are reserved for some purpose, the failure probability based on the left over parity bytes can be computed as $betainc(P_{byte}, 6 - b, (240 - b) - (5 - b)) = betainc(P_{byte}, 6 - b, 240 - 5))$.